Stag Hunt as an Analogy for System-of-systems Engineering

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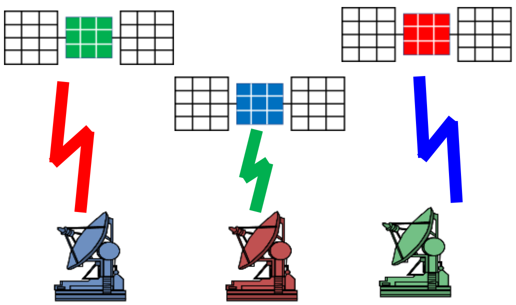
Systems Engineering Trends

- Future engineering systems (in most domains) will be more integrated, connected, and distributed:
  - Internet of Things
  - Cyber-physical Systems
  - Industry 4.0
  - Model-centric Engineering



Federated Satellite Systems

- Systems-of-systems engineering (SoSE) deals with the challenges of coordinating multiple actors
  - Lack of control in conflict with some SE principles
  - Explore indirect approaches to encourage participation

Risk in Distributed Systems

- **Systemic Risk**
  - Uncertainty in cost and schedule
  - Structure: components, interfaces, architecture
  - Dynamics: short- and long-term behavior
  - Technology readiness
- Addressed with existing SE methods/practices

- **Collaborative Risk**
  - Uncertainty in others' actions
  - Conflicting objectives
  - Coordination failure
- Not addressed with existing SE practice
  - Centralized decision-making process
  - Scalar value functions

Research Objective

- Model fundamental coordination dynamics in SoSE using a Stag Hunt problem from game theory
- Highlight the general tradeoff between:
  - Payoff dominance (efficiency)
  - Risk dominance (stability/robustness)
- Generate new insights and perspectives about collective SoSE decision-making and design

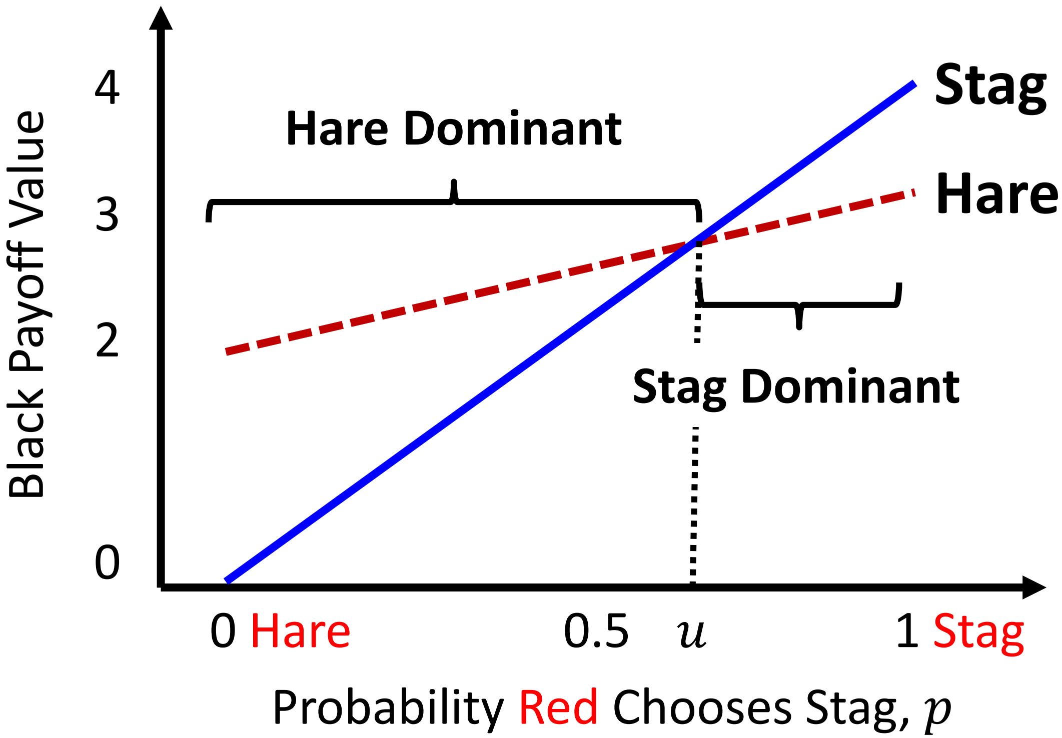
Foundation: Stag Hunt Game

	Hare	Stag
Hare	2 2	0 3
Stag	3 0	4 4

- Two pure Nash equilibria
  - Hare, Hare: risk-dominant equilibrium (minimize risk)
  - Stag, Stag: payoff-dominant equilibrium (maximize reward)

Stag hunt by Gaston Phoebus (Bibliotheque Nationale de France)

Stag Hunt with Uncertainty



	Hare	Stag
Hare	2 / 2	3 / 0
Stag	0 / 3	4 / 4

- $p > u$: choose stag option, $p < u$: choose hare option
- u: Normalized deviation loss, $u = \frac{(2-0)}{(2-0)+(4-3)} = \frac{2}{3}$

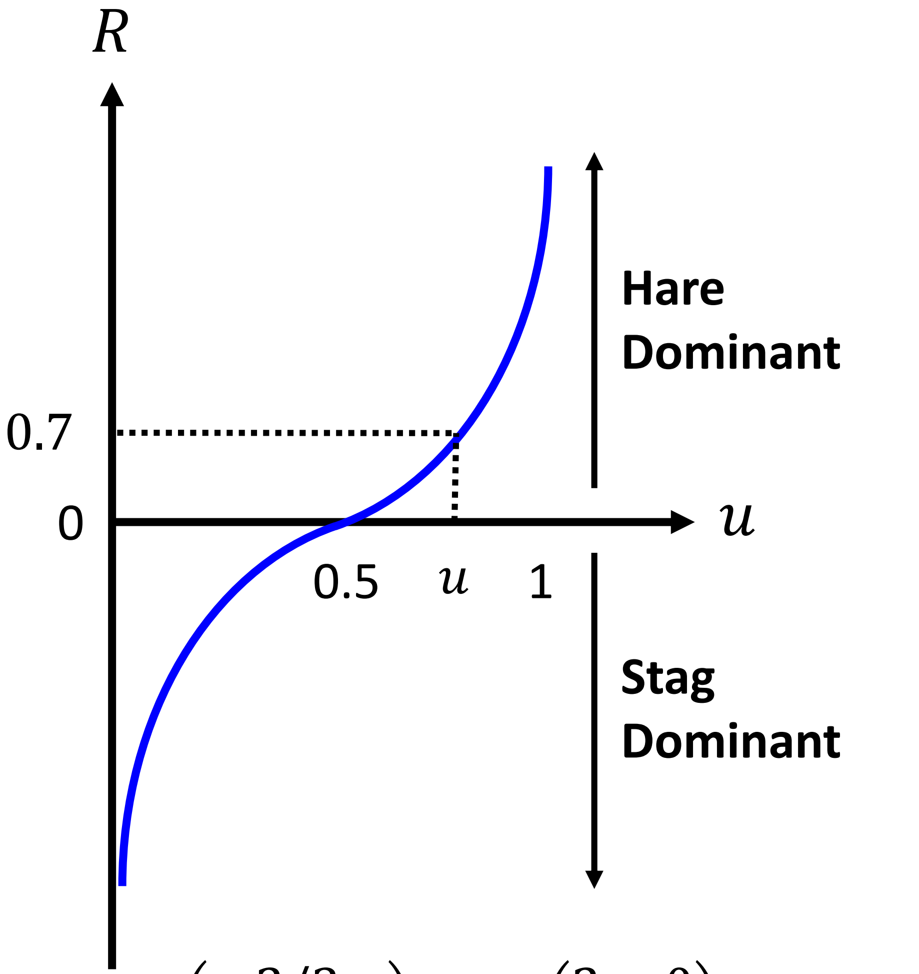
Risk Dominance Metric

- Proposed by Selten (1995) to meet a set of axioms
  - Normative for rational actors
  - Purely objective ($p = 0.5$)
- 2-player asymmetric case:

$$R = \frac{1}{2}\ln\left(\frac{u_1}{1-u_1}\right) + \frac{1}{2}\ln\left(\frac{u_2}{1-u_2}\right)$$

- 2-player symmetric case:

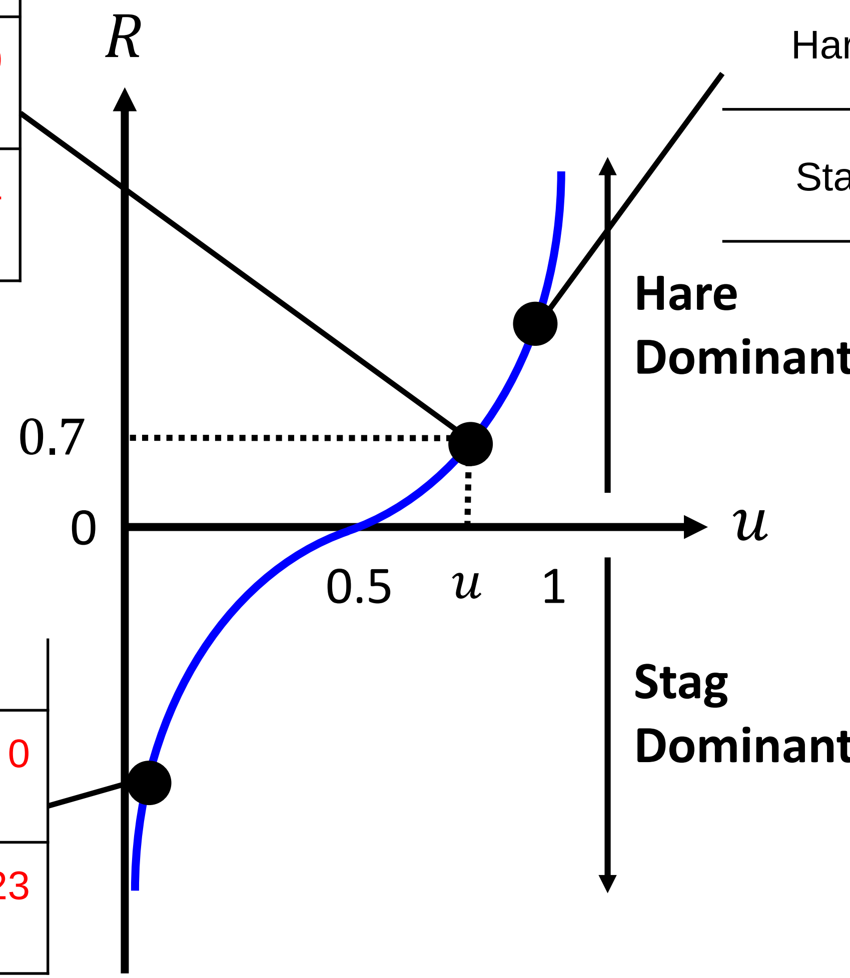
$$R = \ln\left(\frac{u}{1-u}\right)$$



$$R = \ln\left(\frac{2/3}{1-2/3}\right) = \ln\left(\frac{2-0}{4-3}\right) \approx 0.7$$

Comparing Risk Dominance

$R = 0.7$	Hare	Stag
Hare	2 / 2	0 / 3
Stag	3 / 0	4 / 4



$R = 2.3$	Hare	Stag
Hare	2 / 2	-8 / 3
Stag	3 / -8	4 / 4

$R = -2.3$	Hare	Stag
Hare	2 / 2	0 / 3
Stag	3 / 0	23 / 23

Implications for SoSE

- SoSE is like a *strategic design game*
- Each actor chooses a coordination strategy:
  - Remain independent (hare hunter)
  - Join and participate in SoS (stag hunter)
- Values depend on strategy-specific designs:
  - Design space: Axe, Bow, Club, Dog
  - Hare-hunting design (e.g. $d_i^H = D$)
  - Stag-hunting design (e.g. $d_i^S = B$)
- Multi-actor value function: $V_i^{s_1,s_2}\left(d_1^{s_1}, d_2^{s_2}\right)$

SoSE Design Objectives

	Hare	Stag
Hare	$V_2^{HH}(d_1^H, d_2^H)$ $V_1^{HH}(d_1^H, d_2^H)$	$V_2^{HS}(d_1^H, d_2^S)$ $V_1^{HS}(d_1^H, d_2^S)$
Stag	$V_2^{SH}(d_1^S, d_2^H)$ $V_1^{SH}(d_1^S, d_2^H)$	$V_2^{SS}(d_1^S, d_2^S)$ $V_1^{SS}(d_1^S, d_2^S)$

- Select designs to maximize independent value

$$d_1^H = \arg\max_d V_1^{HH}(d, \cdot), \qquad d_2^H = \arg\max_d V_2^{HH}(\cdot, d)$$

- Select designs to maximize (joint) collective value

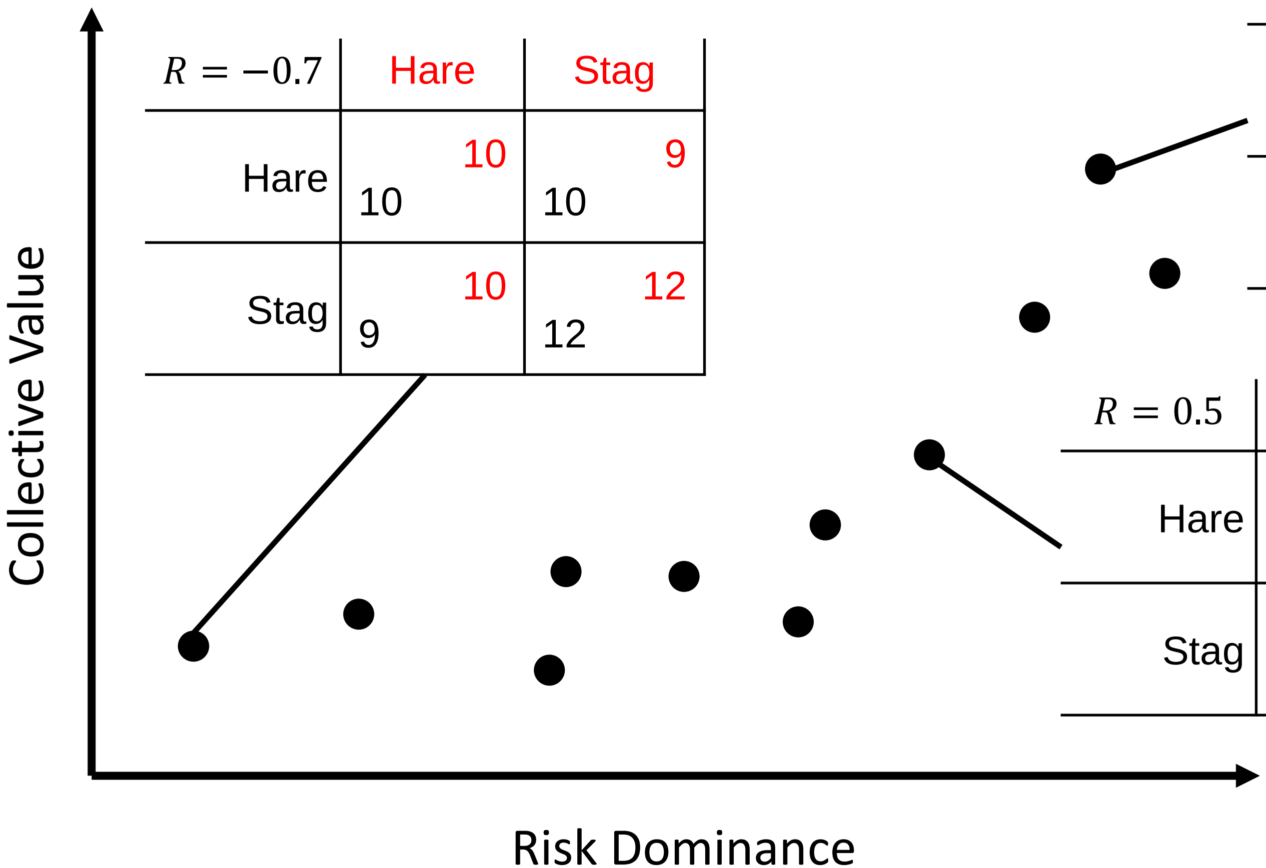
$$d_1^S, d_2^S = \arg\max_{d_1, d_2} V_1^{SS}(d_1, d_2) \cdot V_2^{SS}(d_1, d_2)$$

Considering Risk Dominance

	Hare	Stag
Hare	$V_2^{HH}(d_1^H, d_2^H)$ $V_1^{HH}(d_1^H, d_2^H)$	$V_2^{HS}(d_1^H, d_2^S)$ $V_1^{HS}(d_1^H, d_2^S)$
Stag	$V_2^{SH}(d_1^S, d_2^H)$ $V_1^{SH}(d_1^S, d_2^H)$	$V_2^{SS}(d_1^S, d_2^S)$ $V_1^{SS}(d_1^S, d_2^S)$

- Over-committing to the stag-hunting scenario induces strategic risk from coordination failures
  - Similar to “robust-yet-fragile” behavior
- Mitigate by balancing payoff *and* risk dominance

SoSE Architecture Tradespace



$R = -0.7$	Hare	Stag
Hare	10, 10	10, 9
Stag	9, 10	12, 12

$R = 0.7$	Hare	Stag
Hare	10, 10	10, 0
Stag	0, 10	15, 15

$R = 0.5$	Hare	Stag
Hare	10, 10	10, 5
Stag	5, 10	13, 13

Conclusion

- Fundamental SoSE coordination dynamics modeled using a Stag Hunt problem from game theory
- Equilibria highlight the general tradeoff between:
  - Payoff dominance (efficiency)
  - Risk dominance (stability/robustness)
- SoSE is not simply a joint value optimization problem: must balance sources of strategic risk associated with coordination failures

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